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QUASIGROUPS WITH ORTHOGONAL DISTINCT PARASTROPHES

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Content

1. CONCEPTUAL LANDMARKS OF RESEARCH

The actuality and importance of the topic of thesis. An analogous algebraic notion to that of quasigroup has the origins in the works of Anton K. Suschkewitsch, who published in 1929 papers on the "generalization of the associative law" and studied non-associative binary systems [19]. The notion "quasigroup" was introduced by Ruth Moufang in 1935, who through her studies on Desarguesian planes initiated the development of quasigroup theory as a field of nonassociative algebra [2, 13, 14].

The concept of parastrophe was introduced by A. Sade in the 1950 [15]. An $n-$ary quasigroup has $(n+1)!$ parastrophes, and some or all of them may coincide as algebraic operations. C.C. Lindner and D. Steedley [10] showed that the exact number of distinct parastrophes of a binary quasigroup divides 3! and that there exist binary quasigroups with exactly k distinct parastrophes for each $k = 1, 2, 3$ or 6, giving a complete characterization of the spectrum of finite binary quasigroups with a given exact number of distinct parastrophes. Later, M. McLeish [11] generalized this result to the $n-$ary case, showing that the exact number of distinct parastrophes of an $n-$ary quasigroup divides $(n+1)!$ and studied the existence of ternary quasigroups with a given exact number of distinct parastrophes. Connected to these aspects, the problem of characterizing the spectrum of such n-quasigroups arises. In the ternary case this problem was solved by M. McLeish, completely for finite quasigroups with exactly 1, 3, 4, 6, 12 or 24 distinct parastrophes, and partially for those with 2 or 8 distinct parastrophes [11, 12]. M. McLeish also obtained a series of estimates of the spectrum of quasigroups of arbitrary finite arity n, with a given exact number of distinct parastrophes [11], but the complete characterization of the spectrum is currently an open problem.

One of the approaches used to characterize the spectrum of finite $n-$quasigroups with a given number of distinct parastrophes consists in using, for this purpose, linear quasigroups, in particular $T-$quasigroups. Thus the problem of characterizing linear quasigroups that have an exact given number of distinct parastrophes arises. This problem was solved in the binary case, for linear quasigroups over abelian groups, by G. Belyavskaya and T. Popovich (T. Rotari) [3-6]. It is worth noting that M. McLeish also used linear quasigroups to prove the existence and characterize the spectrum of ternary quasigroups with an exact given number of distinct parastrophes. However, McLeish did not present such characterizations for the case of exactly k distinct parastrophes, for any divisor k of the number 24.

Later, F. Sokhatsky and Y. Pirus [17, 18] obtained characterizations of the ternary quasigroups $(Q, A), A(x_1, x_2, x_3) = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + c$, linear over a group $(Q, +, 0)$, where

0 is the neutral element of the group, α_1, α_2 and α_3 are bijections on the set Q, such that $\alpha_i\ 0 = 0, i = 1, 2, 3$, which possess at most a given number k of distinct parastrophes, where k is a divisor of 24. In particular, F. Sokhatsky and Y. Pirus showed that there are no linear ternary quasigroups (of the given type) with exactly two distinct parastrophes. Note that the results in [17, 18] refer to the case when a ternary quasigroup possesses at most (not exactly) k distinct parastrophes.

In the present thesis, necessary and sufficient conditions are given, such that an $n-$ary $T-$quasigroup $(n = 2, 3, 4)$ possess an exactly given number of distinct parastrophes, and estimatimations of the spectrum of such quasigroups in the finite case are presented.

Another aspect studied in this context concerns the characterization of the spectrum of $n-$ary quasigroups, the distinct parastrophes of which form an orthogonal system. The problem of orthogonality of binary operations initially appeared in combinatorics (orthogonality of Latin squares) being driven by Euler's well-known hypothesis about the non-existence of orthogonal Latin squares of order $n \equiv 2\ (mod\ 4)$ which was definitively (negatively) solved by R. C. Bose, S.S. Shrikhande and E. T. Parker in 1960 [7]. The definitive solution to Euler's conjecture showed that there exist orthogonal Latin squares of any order $q \neq 1, 2, 6$. The process of solving of this hypothesis led to the emergence of new areas of research in combinatorics and algebra, such as, for example, the theory of orthogonal operations, initially approached by T. Mann, V. Belousov, C. Stein, T. Evans, etc. which, requiring various construction methods, developed in multiple directions [8].

A special direction in the theory of orthogonal operations refers to the study of orthogonality of the parastrophes of an $n-$ary quasigroup [1, 20]. $n-$Ary quasigroups that possess orthogonal sets of n parastrophes (principal parastrophes) are called parastrophic-orthogonal (self-orthogonal) quasigroups, and $n-$ary quasigroups, all distinct parastrophes of which form an orthogonal system, are called totally parastrophic-orthogonal quasigroups. The development of methods for constructing parastrophic-orthogonal, respectively totally parastrophic-orthogonal, $n-$ary quasigroups is an efficient tool for characterizing the spectrum of such quasigroups.

A problem that arises in this context is the characterization of linear quasigroups, in particular $T-$quasigroups, which possess an exactly given number of distinct orthogonal parastrophes. An analogous notion to that of a totally parastrophic-orthogonal quasigroup initially appeared in the binary case, being introduced by the author of the thesis in collaboration with G. Belyavskaya, where quasigroups with 6 orthogonal parastrophes were called $totCO-$*quasigroups (totally conjugate - orthogonal quasigroups)* [4].

Parastrophic-orthogonal medial ternary quasigroups were studied by I. Fryz and F. Sokhatsky [9, 16], who established necessary and sufficient conditions for a medial ternary quasigroup to possess orthogonal, respectively strongly orthogonal systems of six (all) principal parastrophes, and proved that, for any $n > 3$, there are no $n-$ary quasigroups whose set of principal parastrophes forms a strongly orthogonal system.

Orthogonal operations, in particular orthogonal (parastrophic-orthogonal, self-orthogonal) quasigroups have numerous applications in cryptography, codes theory, combinatorics, etc. [8].

The purpose and objectives of the thesis. The porpose of the thesis is to obtain characterizations of $n-$ary quasigroups ($n = 2, 3, 4$), which possess an exact given number of distinct parastrophes, in particular of totally parastrophic-orthogonal quasigroups, as well as to estimate the spectrum of these quasigroups. To achieve the purpose of the thesis, the following **objectives** were formulated:

- determining the maximal sets of distinct parastrophes of an n-ary quasigroup ($n = 2, 3, 4$), using the subgroups of the group S_n;
- characterization of the $T-$form of $T-$quasigroups with an exact given number of distinct parastrophes, including the case when they form an orthogonal system;
- obtaining estimations of the spectrum of finite $n-$ary quasigroups ($n = 2, 3, 4$), which have an exact given number of distinct parastrophes, including orthogonal ones.

Scientific novelty and originality. Two new classes of binary quasigroups are introduced in the present thesis: $DC-$ quasigroups (binary quasigroups with six distinct parastrophes) and $totCO-$quasigroups (binary quasigroups whose six parastrophes form an orthogonal system). The problem of the existence of quasigroups possessing a given number of distinct parastrophes and of the characterization of their spectrum, formulated by C.C. Lindner and D. Steedly, is considered for the class of $n-$ary $T-$quasigroups ($n = 2, 3, 4$), including such quasigroups with maximal orthogonal systems of distinct parastrophes.

Important scientific problem solved in the field consists in characterizing binary quasigroups that possess 6 distinct parastrophes, respectively 6 orthogonal parastrophes, in describing binary $T-$quasigroups with exactly 1, 2, 3 or 6 distinct parastrophes, ternary $T-$quasigroups with exactly 3, 4 or 6 distinct parastrophes and $4-$ary $T-$quasigroups with exactly 1, 5, 10 or 20 distinct parastrophes, estimating their spectrum, determining necessary and sufficient conditions for maximal systems of distinct parastrophes to be orthogonal, as well as in proving the non-existence of $4-$ary $T-$quasigroups with exactly 2, 6 or 15 distinct parastrophes.

Theoretical significance and applicative value of the work. The results concerning the $T-$forms of $n-$ary $T-$quasigroups with an exact given number of distinct parastrophes, including orthogonal ones, as well as the proposed methods for constructing parastrophic-orthogonal $n-$quasigroups, represent contributions to the solution of open problems about the existence of $n-$quasigroups with a given number of distinct parastrophes and the spectrum of parastrophic-orthogonal $n-$quasigroups.

Approval of scientific results. The scientific results were presented within eight special sessions of the Seminar " Algebra and Mathematical Logic", dedicated to the memory of Professor Valentin Belousov, held at the "Vladimir Andrunachievich" Institute of Mathematics and Computer Science. Also, the results included in the thesis were presented at 17 specialized conferences, including 7 international conferences outside the Republic of Moldova:

- The 32th International Conference on Applied and Industrial Mathematics, Bucharest, Romania, 18-21 September, 2025;
- The 31th International Conference on Applied and Industrial Mathematics, Oradea, Romania, 19-22 September, 2024;
- The XII International Algebraic Conference in Ukraine dedicated to the 215th anniversary of V. Bunyakovsky, July 02-06, 2019, Vinnytsia, Ukraine;
- The XI International Algebraic Conference in Ukraine dedicated to the 75th anniversary of V. V. Kirichenko, July 03-07, 2017, Kyiv, Ukraine
- The 8-th International Algebraic Conference in Ukraine Dedicated to the memory of Professor V. M. Usenko, Lugansk, 5-12 July, 2011;
- X Международный семинар „Дискретная математика и ее приложения”, Москва, 1-6 февраля 2010;
- The 7-th International Algebraic Conference in Ukraine. Kharkov, 18-23 August, 2009;
- International Conference on Quasigroups and Related Systems (ConfQRS 2025), July 2 – 4, 2025, Chișinău, Republic of Moldova;
- International Conference Mathematics & IT: Research and Education (MITRE– 2025), June 26– 29, 2025, Chișinău, Republic of Moldova;
- International Conference dedicated to the 60th anniversary of the foundation of V. Andrunachievici Institute of Mathematics and Computer Science, October 10-13, 2024, Chișinău, Republic of Moldova;
- International Scientific Conference Mathematics & IT: Research and Education (MITRE-2023), Chișinău, Republic of Moldova, 26 – 29 June, 2023;

- Conference on Applied and Industrial Mathematics dedicated to Academician Mitrofan M. Ciobanu, Chișinău, Republic of Moldova, 22 – 25 August, 2012;
- International Scientific Conference "Mathematics & IT: Research and Education (MITRE-2011), Chișinău, Republic of Moldova, 22 – 25 August, 2011;
- International Scientific Conference "Mathematics & IT: Research and Education" (MITRE-2009), Chișinău, Republic of Moldova, 8 – 9 octombrie, 2009;
- Conferința științifică internațională „Relevanța și calitatea formării universitare: competențe pentru present și viitor european", consacrată aniversării de 80 de ani de la fondarea Universității de Stat „Alecu Russo" din Bălți, 3-4 octombrie 2025;
- National conference with international participation: Natural Sciences in the Dialogue of Generations, State University of Moldova, September 18-19, 2025;
- International Workshop on Intelligent Information System: Proceeding IIS, Chișinău, 13-14 September, 2011.

Publications on the topic of the thesis. A total of 27 scientific papers were published on the topic of the thesis, including 10 articles, of which 6 articles in peer-reviewed scientific journals, 4 articles published in collections of articles (Proceedings) and 17 abstracts of the talks at scientific conferences.

The structure and volume of the thesis. The thesis is written in Romanian and contains: introduction, four chapters, general conclusions and recommendations, bibliography with 162 titles and 5 annexes. The volume of the thesis is of 150 pages, including 112 pages of basic text.

Keywords: $n-$ ary, quasigroup parastrophe, linear quasigroup, $T-$ quasigroup, (totally) parastrophic-orthogonal quasigroup, DC −quasigroup, $totCO$ −quasigroup.

2. THE CONTENT OF THE THESIS

The thesis is structured in four chapters, Introduction, General conclusions and recommendations, Bibliography and 5 annexes.

In the first chapter - **Analysis of the bibliography in the field of quasigroup theory with a given number of distinct parastrophes**, an analysis of the known results that relate to the topic of the thesis is presented. It is shown that the maximum number of distinct parastrophes of a n-ary quasigroup (Q, A) coincides with the index $|S_{n+1}: H|$ of the subgroup $\mathrm{H} = \{\sigma \in \mathrm{S}_{n+1} \mid \mathrm{A} = {}^{\sigma}\mathrm{A}\}$ in the group S_{n+1}. Thus, the maximal possible number of distinct parastrophes of an $n-$ary quasigroup A is a divisor of the number $(n+1)!$ and the maximal sets of distinct parastrophes of the $n-$ary quasigroup A are the sets of representatives of the classes in the quotiont set, obtained when factoring the group S_{n+1} by H.

In this chapter, estimations of the spectrum of binary and, respectively, ternary quasigroups with an exact number of distinct parastrophes are given. It is shown that there exist binary quasigroups that have exactly k distinct parastrophes, for each $k = 1, 2, 3$ or 6, of any order $q \geq 4$. In the ternary case, the results obtained by M. McLeish regarding the existence of ternary and $n-$ary quasigroups with an exact given number of distinct parastrophes and their spectrum are presented [11, 12].

Also, in the first chapter, the results obtained by C.C. Lindner and D. Steedly [10] are presented, which characterize the set of distinct parastrophes of a binary quasigroup (Q, A) using the identities of two variables from the set:

$$T = \{A(x, A(x, y)) = y, \quad A(A(y, x), x) = y, \quad A(x, y) = A(y, x),$$
$$A(x, A(y, x)) = y, \quad A(A(x, y), x) = y\}.$$

In this thesis, the result obtained in [10] is specified, eliminating the penultimate identity from the set T and using for this purpose the set:

$$\bar{T} = \{A(x, A(x, y)) = y, A(A(y, x), x) = y, A(x, y) = A(y, x), A(A(x, y), x) = y\}.$$

Proposition 1.1.4. *Let (Q, A) be a binary quasigroup. The following statements are true:*

1) *if the quasigroup (Q, A) satisfies exactly two identities of the set $\bar{T}$, then all its parastrophes coincide;*
2) *if the quasigroup (Q, A) satisfies the identity $A(A(x, y), x) = y$, then (Q, A) has exactly two distinct parastrophes, and these are $A(x, y)$ and ${}^{(12)}A(x, y)$;*

3) *if the quasigroup* (Q, A) *satisfies exactly one of the identities* $A(x, A(x, y)) = y$, $A(A(y, x), x) = y$, $A(x, y) = A(y, x)$, *then* (Q, A) *has exactly the following three distinct parastrophes* $A(x, y)$, ${}^{(123)}A(x, y)$, ${}^{(132)}A(x, y)$;
4) *if the quasigroup* (Q, A) *does not satisfy any of the identities of the set* $\overline{T}$, *then all its parastrophes are distinct.*

In the last paragraph of the first chapter, a method for constructing orthogonal systems of $n-$ary quasigroups, given by T. Evans, is described, where orthogonal systems of quasigroups of lower arity and the method of superpositions are used.

In the second chapter - **Binary and ternary linear quasigroups with a given maximal number of distinct parastrophes,** the results of the thesis author are presented, regarding binary and, respectively, ternary $T-$quasigroups, with an exact given number of distinct parastrophes.

In paragraph 2.1 the necessary and sufficient conditions are given when a binary $T-$quasigroup has exactly 1, 2 or 3 distinct parastrophes and the spectrum of such quasigroups is characterized.

Proposition 2.1.1. *A binary* $T-$*quasigroup* (Q, A), *with a* $T-$ *group (Q,+), is a* $TS-$*quasigroup if and only if*

$$A(x_1, x_2) = Ix_1 + Ix_2 + c,$$

$\forall\, x_1, x_2 \in Q$, *where* $c \in Q$ *and* $I(x) = -x, \forall x \in Q$.

Corollary 2.1.2. *There exist binary* $TS-$*quasigroups of any order* $q \geq 1$.

Proposition 2.1.2. *A binary* $T-$*quasigroup* (Q, A), *with a* $T-$ *group (Q,+), has exactly two distinct parastrophes if and only if there exist* $\alpha \in Aut(Q, +)$ *and an element* $c \in Q$, *such that*

$$A(x_1, x_2) = \alpha x_1 + a^{-1} x_2 + c,$$

where $\alpha \neq I, \alpha^3 = I, \alpha c = -c$.

Corollary 2.1.4. *There exist binary finite T-quasigroups, which have exactly two distinct parastrophes, of any primary order* $q > 3$.

Proposition 2.1.3. *A binary* $T-$*quasigroup* (Q, A), *with a* $T-$ *group (Q,+), has exactly three distinct parastrophes if and only if there exists* $\alpha \in Aut(Q, +)$, $\alpha \neq I$, *and an element* $c \in Q$, *such that the operation* $A(x_1, x_2)$ *has one of the following three forms:*

$$\alpha x_1 + \alpha x_2 + c, Ix_1 + \alpha x_2 + c, \quad \alpha x_1 + Ix_2 + c.$$

Corollary 2.1.9. *There exist binary finite T-quasigroups, which have exactly three distinct parastrophes, of any order* $q > 2$.

The Paragraph 2.2 refers to binary quasigroups possessing six (all) distinct parastrophes, called DC −quasigroups, and the following basic results are presented.

Proposition 2.2.1. *Let (Q, A) be a DC −quasigroup. The following statements hold:*

1) Any DC −quasigroup is noncommutative and nontrivial.

2) Any quasigroup that contains a DC −subquasigroup is a DC −quasigroup.

3) Any parastrophe of a DC −quasigroup is a DC −quasigroup.

4) Any nontrivial quasigroup that is a homomorphic image of a DC − quasigroup is a DC −quasigroup.

C.C. Lindner and D. Steedly [10], showed that there exist finite binary quasigroups, with all six distinct parastrophes, of any order $q \geq 4$. The following statements present a criterion when a binary T −quasigroup is a DC −quasigroup and the characterization of the spectrum of finite $DC - T$ −quasigroups.

Theorem 2.2.1. *A T −quasigroup (Q, A), $A(x,\ y) = \varphi x + \psi y$, is a DC −quasigroup if and only if $\varphi \neq I, \psi$; $\psi \neq\ I$ and $\varphi^2 \neq I\psi$ or $\psi^2 \neq I\varphi$.*

Theorem 2.2.2. *There exist $DC - T$ −quasigroups of order q, for every $q \geq 5,\ q \neq 6$.*

The spectrum of finite ternary quasigroups, with exactly k distinct parastrophes, where k is a divisor of the number 24, was characterized completely in the case $k = 1, 3, 4, 6, 12, 24$ and partially in the case $k = 2$ or 8, by M. McLeish [11, 12].

F. Sokhatsky and Y. Pirus in [17, 18] obtained characterizations of ternary quasigroups (Q, A), $A(x_1, x_2, x_3) = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + c$, linear over a group $(Q, +, 0)$, where 0 is the neutral element of the group, α_i, $i = 1, 2, 3$, are bijections on Q, such that $\alpha_i 0 = 0, i = 1, 2, 3$, which has at most k distinct parastrophes (not exactly k distinct parastrophes), where k is a divisor of 24.

In paragraph 2.3 necessary and sufficient conditions are given for a ternary T −quasigroup to possess exactly k distinct parastrophes, where $k = 3$, 4 or 6. Also, in this paragraph characterizations of the spectrum of such ternary quasigroups are given.

Proposition 2.3.2. *A ternary T −quasigroup (Q, A), with a T −group $(Q, +)$, has exactly three distinct parastrophes if and only if there exist $\alpha \in Aut(Q, +)$ and an element $c \in Q$, such that the operation $A(x_1, x_2, x_3)$ has one of the following three forms: $\alpha x_1 + \alpha x_2 + I x_3 + c, I x_1 + \alpha x_2 + \alpha x_3 + c$, $\alpha x_1 + I x_2 + \alpha x_3 + c$, where $\alpha \neq I$, $\alpha^2 = \varepsilon$, $\alpha c = Ic, I(x) = -x$, $\forall x \in Q$.*

Corollary 2.3.1. *There exist ternary idempotent $T-$quasigroups with exactly three distinct parastrophes.*

Corollary 2.3.2. *For any $m \geq 3$ there exist ternary quasigroups of order m with exactly three distinct parastrophes.*

Using the subgroups of order six of the group S_4, $T-$forms of ternary $T-$quasigroups with exactly four distinct parastrophes, and their spectrum, are characterized in paragraph 2.3.

Theorem 2.3.1. *A ternary $T-$quasigroup (Q, A), with a $T-$group $(Q,+)$, has exactly four distinct parastrophes if and only if there exist $\alpha \in Aut(Q,+)$ and an element $c \in Q$, such that its $T-$form i sone of the following:*

$$T_1 = \big((Q,+),\alpha,\alpha,\alpha,c\big), \quad T_2 = \big((Q,+),I,I,\alpha,c\big),$$
$$T_3 = \big((Q,+),I,\alpha,I,c\big), \quad T_4 = \big((Q,+),\alpha,I,I,c\big),$$

where $Ix = -x$, $\alpha \neq I$.

Corollary 2.3.4. [11] *There are finite ternary quasigroups of any odd order $q \geq 3$ which have exactly 4 distinct parastrophes.*

Also, in paragraph 2.3 of the thesis, ternary quasigroups with exactly six distinct parastrophes are studied, using the seven subgroups of order four of the group S_4. For each subgroup, the $T-$form of the corresponding $T-$quasigroup was determined and the necessary and sufficient conditions for a ternary $T-$quasigroup to possess exactly six distinct parastrophes were established. A series of estimations of the spectrum of these quasigroups have been obtained.

Theorem 2.3.2. *A ternary $T-$quasigroup (Q,A) with the T-group $(Q,+)$ and such that $H = \{\sigma| \,^{\sigma}A = A\} \cong Z_4$, has exactly six distinct parastrophes if and only if there exist $\alpha \in Aut(Q,+)$ and an element $c \in Q$, such that its $T-$form i sone of the following:*

$$T_1 = \big((Q,+),\alpha,I\alpha^2,\alpha^3,c\big), T_2 = \big((Q,+),I\alpha^2,\alpha^3,\alpha,c\big), \ T_3 = \big((Q,+),\alpha,\alpha^3,I\alpha^2,c\big),$$

where $\alpha c = Ic, Ix = -x, \alpha^2 \neq \varepsilon, \alpha^4 = \varepsilon$.

Theorem 2.3.3. *A ternary $T-$quasigroup (Q,A) with the T-group $(Q,+)$ and such that $H = \{\sigma| \,^{\sigma}A = A\} = H_i$, $i = 18, 19, 20$, has exactly six distinct parastrophes if and only if there exist $\alpha \in Aut(Q,+)$ and an element $c \in Q$, such that its $T-$form is sone of the following: $T_4 = \big((Q,+),\alpha,\alpha,I,c\big)$, $T_5 = \big((Q,+),\alpha, I,\alpha,c\big), T_6 = \big((Q,+),I,\alpha,\alpha,c\big)$, where $\alpha^2 \neq \varepsilon$.*

Theorem 2.3.4. *A ternary $T-$quasigroup (Q,A) with the T-group $(Q,+)$ and such that $H = \{\sigma| \,^{\sigma}A = A\} = H_{21} = K_4$, has exactly six distinct parastrophes if and only if there exist $\alpha,\beta \in$*

$Aut(Q,+)$ *and an element* $c \in Q$, *such that its* T *−form is* $T_7 = \big((Q,+), I\alpha\beta, \alpha, \beta, c\big)$, *where* $\alpha c = \beta c = Ic$, $\alpha^2 = \beta^2 = \varepsilon, \alpha\beta = \beta\alpha$, $\alpha \neq \beta, \alpha \neq I, \beta \neq I$.

Corollary 2.3.6. *There exist finite ternary quasigroups of any odd order* $q, (q, 3) = 1$, *which possesses exactly six distinct parastrophes.*

Chapter three concerns with **4-ary T-quasigroups with a given exact number of distinct parastrophes**. Necessary and sufficient conditions are given for a 4 −ary T −quasigroup to have exactly 1, 5, 10, or 20 distinct parastrophes. Some estimates of the spectrum of such quasigroups are presented.

Proposition 3.1.3. *A* $4 - T$ *−quasigroup (Q, A) is a* $4 - TS - T$ *−quasigroup, if and only if it has the* T *−form* $\big((Q,+), I, I, I, I, c\big)$, *where* $Ix = -x$, $\forall x \in Q$.

Corollary 3.1.1. *There exist* $4 - TS - T$ *−quasigroups of any order* $q \in \mathbb{N}, q \geq 2$.

4-Ary quasigroups with exactly five distinct parastrophes are characterized by the subgroups of order 24 of the group S_5. The symmetric group S_5 has the following 5 subgroups of order 24, all isomorphic to S_4:

$H_1 = \langle(1234), (12)\rangle$, $H_3 = \langle(1245), (12)\rangle$, $H_5 = \langle(2345), (23)\rangle$.
$H_2 = \langle(1235), (12)\rangle$, $H_4 = \langle(1345), (13)\rangle$,

Proposition 3.3.1. *A 4-ary* T *−quasigroup* (Q, A), *with the T-group* $(Q,+)$, *has exactly five distinct parastrophes if and only if there exist* $\alpha \in Aut(Q,+)$ *and an element* $c \in Q$, *such that its* T *− form is sone of the following:* $T_1 = \big((Q,+), \alpha, \alpha, \alpha, \alpha, c\big)$, $T_2 = (Q,+), I, I, I, \alpha, c)$, $T_3 = \big((Q,+), I, I, \alpha, I, c\big)$, $T_4 = \big((Q,+), I, \alpha, I, I, c\big)$, $T_5 = \big((Q,+), \alpha, I, I, I, c\big)$,*where* $\alpha \neq I$, $Ix = -x$.

Corollary 3.3.2. There e*xist* $4 - T$ *−quasigroups of order q, with exactly 5 distinct parastrophes, for any* $q \geq 3$.

Let (Q, A) be a 4 −ary quasigroup and $H = \{\sigma \in S_5 \mid {}^{\sigma}A = A\}$, such that $|H| = 12, H \leq S_5$. The group S_5 has 15 such subgroups, 5 of which are isomorphic with the altern group A_4, and 10 subgroups are isomorphic with $S_2 \times S_3$.

Proposition 3.4.1. *There does not exist* $4 - T$ *−quasigroups* (Q, A) *with 10 distinct parastrophes, such that* $H = \{\sigma \in S_5 \mid {}^{\sigma}A = A\} \cong A_4$.

Proposition 3.4.2. *Let (Q, A) be a* $4 - T$ *−quasigroup with the* T *−group (Q, +) and let* $H \in \{H_i, i = \overline{6,15}\}$, *where* $H = \{\sigma \in S_5 \mid {}^{\sigma}A = A\} \cong S_2 \times S_3$. *Then there exist* $\alpha \in Aut(Q,+)$ *and an element* $c \in Q$, *where* $\alpha \neq I, Ix = -x, \forall x \in Q$, *0 is the neutral element of* $(Q,+)$, *such that (Q, A) ha sone of the following* T *−forms:*

$$T_1 = ((Q,+),\alpha,\alpha,\alpha,I,c), \quad T_2 = ((Q,+),\alpha,\alpha,I,\alpha,c),$$
$$T_3 = \big((Q,+),\alpha,I,\alpha,\alpha,c\big), \quad T_4 = ((Q,+),I,\alpha,\alpha,\alpha,c),$$
$$T_5 = ((Q,+),I,I,\alpha,\alpha,c), \quad T_6 = ((Q,+),I,\alpha,I,\alpha,c),$$
$$T_7 = ((Q,+),I,\alpha,\alpha,I,c), \quad T_8 = ((Q,+),\alpha,I,I,\alpha,c),$$
$$T_9 = ((Q,+),\alpha,I,\alpha,I,c), \quad T_{10} = ((Q,+),\alpha,\alpha,I,I,c).$$

Corollary 3.4.2. *For any odd* q, $q \geq 3$ *there exist* $4-T$ *−quasigroups with exactly 10 distinct parastrophes.*

To obtain the characterization of the T −forms of $4-T$ −quasigroups with exactly 20 distinct parastrophes, we consider all 30 subgroups of order 6 of the group S_5, namely:

1) 10 subgroups isomorphic to the cyclic group Z_6:

$$H_1 = \langle(123)(45)\rangle, H_2 = \langle(124)(35)\rangle, \; H_3 = \langle(125)(34)\rangle, \; H_4 = \langle(134)(25)\rangle,$$
$$H_5 = \langle(135)(24)\rangle, \quad H_6 = \langle(145)(34)\rangle, \quad H_7 = \langle(234)(15)\rangle,$$
$$H_8 = \langle(235)(14)\rangle, \quad H_9 = \langle(245)(13)\rangle, \quad H_{10} = \langle(345)(12)\rangle;$$

2) 20 subgroups izomorphic to S_3, which are generated by two substitutions α and β, of order 3 and, respectively 2, including:

2a) 10 subgroups, where β is a transposition:

$$H_{11} = \langle(123),(12)\rangle, \; H_{12} = \langle(124),(12)\rangle, \; H_{13} = \langle(134),(13)\rangle,$$
$$H_{14} = \langle(125),(12)\rangle, \quad H_{15} = \langle(135),(13)\rangle, \; H_{16} = \langle(145),(14)\rangle,$$
$$H_{17} = \langle(234),(23)\rangle, \quad H_{18} = \langle(235),(23)\rangle, H_{19} = \langle(245),(24)\rangle,$$
$$H_{20} = \langle(345),(34)\rangle;$$

2b) 10 subgroups, where β is a product of two independent transpositions:

$$H_{21} = \langle(123),(12)(45)\rangle, \; H_{22} = \langle(124),(12)(35)\rangle, \; H_{23} = \langle(125),(12)(34)\rangle,$$
$$H_{24} = \langle(134),(13)(25)\rangle, \quad H_{25} = \langle(135),(13)(24)\rangle, \; H_{26} = \langle(145),(14)(23)\rangle,$$
$$H_{27} = \langle(234),(23)(15)\rangle, \quad H_{28} = \langle(235),(23)(14)\rangle, \quad H_{29} = \langle(245),(13)(24)\rangle,$$
$$H_{30} = \langle(345),(34)(12)\rangle.$$

The following results were obtained:

Theorem 3.6.1. *Let (Q, A) be a* $4-T$ *−quasigroup with the* T *−group* $(Q,+)$ *and let* $H \in \{H_i, i = \overline{11,20}\}$, *where* $H = \{\sigma \in S_5 \mid {}^{\sigma}A = A\}$. *Then there exist* $\alpha, \beta \in Aut(Q,+)$ *and an element* $c \in Q$, *such that* $\alpha \neq \beta$, $\alpha \neq I$, $\beta \neq I$, $\alpha c + c \neq 0$, $\beta c + c \neq 0$, $Ix = -x$, $\forall x \in Q$, *where 0 is the neutral element of* $(Q,+)$, *such that* (Q,A) *has one of the following* T *−forms:*

$$T_1 = \big((Q,+),\alpha,\alpha,\alpha,\beta,c\big), \quad T_2 = \big((Q,+),\alpha,\alpha,\beta,\alpha,c\big), \quad T_3 = ((Q,+),\alpha,\beta,\alpha,\alpha,c),$$
$$T_4 = ((Q,+),\beta,\alpha,\alpha,\alpha,c), \quad T_5 = ((Q,+),I,I,\alpha,\beta,c), \quad T_6 = ((Q,+),I,\alpha,I,\beta,c),$$
$$T_7 = \big((Q,+),I,\alpha,\beta,I,c\big), \quad T_8 = \big((Q,+),\alpha,I,I,\beta,c\big), \quad T_9 = ((Q,+),\alpha,I,\beta,I,c),$$

$$T_{10} = ((Q,+),\alpha,\beta,I,I,c).$$

Proposition 3.6.1. *The* $4-T$ *−quasigroup* (Q,A) *with the* T *−form* $T_1 = \big((Q,+),\alpha,\alpha,\alpha,\beta,c\big)$*, where* $\alpha \neq \beta, \alpha \neq I, \beta \neq I, I(x) = -x, \forall x \in Q$*, has exactly 20 distinct parastrophes.*

Corollary 3.6.1. *A* $4-T$ *−quasigroup (Q, A), where* $\{\sigma \in S_5 |\ A = {}^{\sigma}A\} = \langle(123),(23)\rangle$*, has exactly 20 distinct parastrophes if and only if there exist* $\alpha,\beta \in Aut(Q,+)$ *and an element* $c \in Q$*, such that (Q, A) has the* $T-$ *form* $((Q,+),\alpha,\alpha,\alpha,\beta,c)$*, where* $\alpha \neq \beta, \alpha \neq I, \beta \neq I, I(x) = -x, \forall x \in Q$.

Theorem 3.6.2. *There does not exist* $4-T$ *−quasigroups (Q, A), such that the group* $H = \{\sigma \in S_5 |\ A = {}^{\sigma}A\}$ *is isomorphic to* $\mathbb{Z}_6$.

Theorem 3.6.3. *There does not exist* $4-T$ *−quasigroups (Q,A), such that* $H \in \{H_{21}, H_{22}, \ldots, H_{30}\}$*, where* $H = \{\sigma \in S_5 |\ A = {}^{\sigma}A\}$.

Corollary 3.6.2. *There exist* $4-T$ *−quasigroups with exactly 20 distinct parastrophes of any odd order* $q > 1$*, where* $(q,3) = 1$.

Also, in chapter three it is proved that there are no 4−ary T −quasigroups with exactly 2, 6 or 15 distinct parastrophes.

The group S_5 has only one subgroup of order 60 and this is A_5. The two distinct parastrophes of a 4 −ary quasigroup (Q,A) with $\{\sigma \in S_5 \mid {}^{\sigma}A = A\} = A_5$, are given by sets of representatives of the cosets $\{A_5,\ A_5\tau \mid \tau \in S_5 \setminus A_5\}$.

Proposition 3.2.1. *There does not exist* $4-T-$ *quasigroups with exactly two distinct parastrophes.*

$4-$ Ary quasigroups with exactly six distinct parastrophes are characterized by the subgroups of order 20 of the group S_5. This group has exactly 6 such subgroups, namely:

$H_1 = \langle(12345),(2453)\rangle$, $H_2 = \langle(12435),(2453)\rangle$, $H_3 = \langle(12453),(2435)\rangle$,
$H_4 = \langle(12543),(2534)\rangle$, $H_5 = \langle(12534),(2543)\rangle$, $H_6 = \langle(13524),(3542)\rangle$.

Proposition 3.2.2. *There does not exist* $4-T-$ *quasigroups with exactly six distinct parastrophes.*

$4-$ Ary quasigroups with exactly 15 distinct parastrophes are characterized by the subgroups of order 8. The group S_5 has a total of 15 such subgroups, all being isomorphic to the group D_8:

$H_1 = \langle(1234),(13)\rangle$; $H_3 = \langle(1324),(12)\rangle$; $H_5 = \langle(1253),(15)\rangle$;
$H_2 = \langle(1243),(14)\rangle$; $H_4 = \langle(1235),(13)\rangle$; $H_6 = \langle(1325),(12)\rangle$;

$H_7 = \langle(1245), (14)\rangle$;	$H_{10} = \langle(1345), (14)\rangle$;	$H_{13} = \langle(2345), (24)\rangle$;
$H_8 = \langle(1254), (15)\rangle$;	$H_{11} = \langle(1354), (15)\rangle$;	$H_{14} = \langle(2354), (25)\rangle$;
$H_9 = \langle(1425), (12)\rangle$;	$H_{12} = \langle(1435), (13)\rangle$;	$H_{15} = \langle(2435), (23)\rangle$.

Proposition 3.5.1. *There does not exist $4 - T -$quasigroups with exactly 15 distinct parastrophes.*

Chapter 4 deals with parastrophic$-$orthogonal $n-$ary quasigroups, including totally parastrophic$-$orthogonal ones, for $n = 2, 3, 4$.

In the binary case, we obtained characterizations of quasigroups in which all six parastrophes form an orthogonal system, called $totCO$ $-$quasigroups. It is shown that the class of $totCO$ $-$quasigroups is closed under the parastrophy transformation and homomorphic images, necessary and sufficient conditions are given when a binary $T-$ quasigroup is a $totCO$ $-$quasigroup. Estimations of the of the $totCO$ $-$quasigroups spectrum are given.

Proposition 4.1.1. A *$T-$ quasigroup (Q, A), where $A(x, y) = \varphi x + \psi y + c, c \in Q$, is a $totCO$ $-$quasigroup if and only if the mappings $\varphi + \varepsilon$, $\varphi - \varepsilon$, $\psi + \varepsilon$, $\psi - \varepsilon$, $\varphi^2 + \psi$, $\psi^2 + \varphi$, $\varphi - \psi$, $\varphi + \psi$, $\psi\varphi - \varepsilon$ are bijections.*

Corollary 4.1.1. *A T $-$quasigroup $(\mathbb{Z}_n, A)$, $A(x, y) = \bar{a}x + \bar{b}y$, is a $totCO$ $-$quasigroup if anly if the numbers $a + 1$, $a - 1$, $b + 1$, $b - 1$, $a^2 + b$, $b^2 + a$, $a - b$, $a + b$, $ab - 1$ are mutually prime with n.*

Corollary 4.1.2. *If a T $-$quasigorup $(\mathbb{Z}_n, A)$, $A(x, y) = \bar{a}x + \bar{b}y$, is a $totCO -$ quasigroup, then $a \neq -1, 1, b, -b, -b^2$; $b \neq -1, 1, -a^2$, $ab \neq 1$ $(mod\ n)$.*

Theorem 4.1.1. *For any integer $n \geq 11$, what is reciprocally prime with 2, 3, 5 and 7, there exist $totCO$ $-$quasigroups of order n.*

Corollary 4.1.3. *There exist $totCO$ $-$quasigroups of any order $n = p_1^{k_1} p_2^{k_2} \dots p_s^{k_s}$, where p_i is a prime, $p_i \neq 2, 3, 5, 7$, $k_i \geq 1, i = 1,2, \dots, s$, $s \geq 1$.*

Proposition 4.1.2. *Let consider the quasigroups $(\mathbb{Z}_n, A_i)$, $i = 1,2, \dots, 8$, where:*

$$A_1(x, y) = 2x + 4y, A_2(x, y) = 3x + 5y, A_3(x, y) = 2x + 3y, A_4(x, y) = 2x + 8y,$$
$$A_5(x, y) = 5x + 10y, A_6(x, y) = 5x + 11y, A_7(x, y) = 3x + 7y, A_8(x, y) = 3x + 9y.$$

The following statements are true:

1) The quasigorups $(\mathbb{Z}_n, A_1)$ and $(\mathbb{Z}_n, A_2)$, are $totCO$ $-$quasigroups if and only if n is mutually prime with each of the numbers 2, 3, 5 and 7;

2) The quasigorups $(\mathbb{Z}_n, A_i)$, where $i = 3,4,5,6$, are $totCO$ $-$quasigroups if and only if n is mutually prime with each of the numbers 2, 3, 5, 7 and 11;

3) The quasigorups $(\mathbb{Z}_n, A_7)$ *and* $(\mathbb{Z}_n, A_8)$, *are* $totCO$ *−quasigroups if and only if* n *is mutually prime with each of the numbers 2, 3, 5, 7 and 11.*

Proposition 4.1.3. *Any parastrophe of a* $totCO$ *−quasigroup is a* $totCO$ *−quasigroup.*

Necessary and sufficient conditions are given for a ternary T-quasigroup to have exactly three or exactly four distinct parastrophes, that form an orthogonal system. Estimations of the spectrum of such quasigroups are obtained.

Theorem 4.2.1. A ternary T *−quasigroup* (Q, A), *with the* T *−group* $(Q, +)$, *has exactly three distinct parastrophes that are also orthogonal if and only if the operation* $A(x_1, x_2, x_3)$ *has one of the following forms:*

$$\alpha x_1 + \alpha x_2 + I x_3 + c, \qquad I\ x_1 + \alpha x_2 + \alpha x_3 + c, \qquad \alpha x_1 + I x_2 + \alpha x_3 + c,$$

where $Ix = -x,\ \alpha \neq I, 2\alpha \neq \varepsilon,\ \alpha^2 = \varepsilon,\ \alpha c = Ic,\ \alpha \in\ Aut\ (Q, +),\ c \in Q$.

Corollary 4.2.2. *There exist finite ternary quasigroups, with exactly three distinct parastrophes, that are also orthogonal, of any odd order* $q \geq 3$.

Corollary 4.2.3. *Let* $(\mathbb{R}, +, \cdot)$ *be the field of real numbers and let* $(\mathbb{R}, A)$ *be a ternary quasigroup linear over* $\mathbb{R}$. *Then* $(\mathbb{R}, A)$ *is an idempotent quasigroup with exactly three distinct parastrophes, which are also orthogonal, if and only if the operation* $A(x_1,\ x_2,\ x_3)$ *has one of the forms:*

$$x_1 + x_2 - x_3,\ \ x_1 - x_2 + x_3 \ or\ -x_1 + x_2 + x_3.$$

Corollary 4.2.4. *There exist infinite ternary quasigroups with exactly three distinct parastrophes that are also orthogonal.*

In paragraph 2.3, ternary T −quasigroups with exactly 4 distinct parastrophes are studied, giving the sets of identities that ensure the given property. Also, the T −form of the ternary T −quasigroup operation, which possesses exactly 4 distinct parastrophes, is analyzed. Note that, in this case, a set of four distinct parastrophes of the ternary quasigroup is: $\left\{A, {}^{(12)(34)}A, {}^{(13)(24)}A, {}^{(14)(23)}A\right\}$.

According to Theorem 2.3.1, a ternary T −quasigrup (Q, A), with the T −group $(Q, +)$, has exactly four distinct parastrophes if and only if the operation A has one of the forms: $A_1(x_1,\ x_2, x_3)\ = \alpha x_1 + \alpha x_2 + \alpha x_3, A_2(x_1,\ x_2,\ x_3)\ =\ I x_1 + \alpha x_2 + I x_3,\ A_3(x_1,\ x_2,\ x_3)\ =\ I x_1 + I x_2 + \alpha x_3,\ A_4(x_1, x_2, x_3)\ =\ \alpha x_1 + I x_2 + I x_3$, where $\alpha \in\ Aut\ Q(+)\ \ Ix = -x,\ \alpha \neq I$. Using this result, the following theorem is proved:

Theorem 4.2.2. *A ternary* T *−quasigrup* (Q, A), *with the* T *−group* $(Q, +)$, *with exactly four distinct parastrophes, is totally parastrophic-orthogonal if and only if* $\alpha + \varepsilon,\ \varepsilon + 2I\alpha \in AutQ(+)$, *where* $\alpha \in\ Aut\ Q(+), Ix = -x,\ \alpha \neq I$.

Corollary 4.2.6. *There exist finite ternary quasigroups of any odd order* $q \geq\ 3$ *which have exactly 4 distinct parastrophes that are also orthogonal.*

According to Proposition 3.3.1, a $4 - T$ −quasigroup has exactly five distinct parastrophes if and only if its T −form is one of the following:

$$T_1 = \big((Q,+), \alpha, \alpha, \alpha, \alpha, c\big), \qquad T_2 = (Q,+), I, I, I, \alpha, c), \qquad T_3 = ((Q,+), I, I, \alpha, I, c),$$

$$T_4 = ((Q,+), I, \alpha, I, I, c),\ \ T_5 = ((Q,+), \alpha, I, I, I, c),$$

where $\alpha \neq I,\ Ix = -x$. A necessary and sufficient condition when the five parastrophes are also orthogonal is the following:

Theorem 4.3.1. *A* 4 *−ary* T *−quasigroup (Q, A) with the* T *−form* $\big((Q,+), \alpha, \alpha, \alpha, \alpha, c\big)$, *with exactly five distinct parastrophes, is totally parastrophic-orthogonal if and only if* $\alpha + \varepsilon \in Aut(Q,+)$.

Corollary 4.3.1. *The* 4 *−ary* T *−quasigroup* $((\mathbb{Z}_n$, *A) with the* T *−form* $((\mathbb{Z}_n,+), \bar{a}, \bar{a}, \bar{a}, \bar{a}, \bar{c}\,)$, *where* $(a, n) = 1$ *and* $(a+1, n) = 1$, *has exactly 5 distinct parastrophes that are also orthogonal.*

Corollary 4.3.2. *There exist finite* 4 *−ary* T *−quasigroups, with exactly 5 distinct and orthogonal parastrophes, of any order* $q \geq 3$.

In the last paragraph of Chapter 4, based on the method of constructing orthogonal systems of n −ary operations, given by Trevor Evans, constructions of parastrophic-orthogonal k −ary operations, in particular self-orthogonal ones, are presented, where k has one of the following forms: $k = n^2, 2^n, mn$, for $n, m \geq\ 2$.

Proposition 4.4.1. *Let* (Q, A) *be a self-orthogonal* n *−ary quasigroup and let* $\{{}^{\alpha_1}A, {}^{\alpha_2}A, \dots, {}^{\alpha_n}A\}$ *be an orthogonal system of principal parastrophes of the quasigroup* (Q, A). *The* n^2 *−ary grupoid* (Q, B_1), *where*

$$B_1\big(x_1^{n^2}\big) = {}^{\alpha_1}A\left({}^{\alpha_1}A(x_1^n), {}^{\alpha_1}A(x_{n+1}^{2n}), \dots, {}^{\alpha_1}A\left(x_{n(n-1)+1}^{n^2}\right)\right)$$

is a self-orthogonal quasigroup.

Corollary 4.4.1. *If there exist* n *−ary self-orthogonal quasigroups of order* q, *then there exist* n^2 *−ary self-orthogonal quasigroups of order* q.

Corollary 4.4.2. *There exist 2^n-ary self-orthogonal quasigroups of any order q, where $q \in \mathbb{N}^*\backslash\{1, 2, 3, 6\}$.*

Theorem 4.4.2. *Let (Q, A) and (Q, B) be finite self-orthogonal n −ary, and respectively m −ary, quasigroups. If $\{^{\alpha_1} A, ^{\alpha_2}A, \dots . ^{\alpha_n}A\}$ and $\{^{\beta_1}B, ^{\beta_2}B, \dots, ^{\beta_m}B\}$ are orthogonal systems of principal parastrophes of the respective quasigroups, then the groupoid (Q, C_1), where*

$$C_1(x_1^{mn}) = {}^{\beta_1}B(^{\alpha_1}A(x_1^n), \dots , \quad ^{\alpha_1}A(x_{n(m-1)+1}^{mn}))$$

is a self-orthogonal nm-ary quasigroup.

Corollary 4.4.6. *If on a finite set Q there exist self-orthogonal m−ary and n−ary quasigroups $(n, m \geq 2)$, then on this set there exist self−orthogonal mn−ary quasigroups.*

Corollary 4.4.7. If *there exist self−orthogonal n−ary quasigroups of order q, then there exist self-orthogonal n^k −ary quasigroups of order q, where $k \geq 2$.*

Corollary 4.4.8 *There exist self−orthogonal 2^k-ary quasigroups of any order $q \neq 1, 2, 3, 6$, for every $k \geq 1$.*

Corollary 4.4.9. *There exist self−orthogonal p^k-ary quasigroups of order p for any odd prime p, and any $k \in \mathbb{N}^*$.*

The thesis contains 5 annexes that include all subgroups of the group S_4 and the subgroups of orders 24, 20, 12, 8 and 6, respectively, of the group S_5, T −forms of binary T −quasigroups with exactly k distinct and orthogonal parastrophes, where $k \in \{1, 2, 3, 6\}$, T −forms of ternary T −quasigroups with exactly k distinct parastrophes, where $k \in \{1, 2, 3, 4, 6\}$, all parastrophes of a ternary and, respectively 4 −ary, T −quasigroup.

3. GENERAL CONCLUSIONS AND RECOMMENDATIONS

This thesis refers to the theory of $n-$ary quasigroups with an exact number of distinct parastrophes, in particular to $n-$ary quasigroups with maximal orthogonal systems of distinct parastrophes, called totally parastrophic-orthogonal quasigroups.

The porpose of the thesis is to obtain characterizations of $n-$ary quasigroups ($n = 2, 3, 4$), which possess an exact possible number of distinct parastrophes, in particular, maximal orthogonal sets of parastrophes, as well as to estimate their spectrum.

The problem of the existence of quasigroups, possessing a given number of distinct parastrophes, and of the characterization of their spectrum, formulated by Lindner and Steedly, is considered for the class of $n - T -$ quasigroups ($n = 2, 3, 4$), including the study of maximal orthogonal systems of distinct parastrophes. In the ternary case, the problem of the existence of finite quasigroups with an exact number k of distinct parastrophes was solved by M. McLeish, integrally for $k = 1, 3, 4, 6, 12, 24$ distinct parastrophes and partially for $k = 2, 8$ distinct parastrophes. The problem of characterizing linear binary quasigroups with an exactly given number of distinct parastrophes, including orthogonal ones, was initiated by the author of the thesis in collaboration with G. Beleavskaya [4], and in the ternary case was considered also by F. Sokhatsky, Y. Pirus and I. Fryz.

The thesis containes the following ***main results*** obtained by the author:

1. Two new classes of binary quasigroups are introduced and studied: $DC -$ quasigroups (quasigroups with six distinct parastrophes) and $totCO -$ quasigroups (quasigroups with six orthogonal parastrophes);
2. It is proved that the class of $DC -$ quasigroups is closed under the parastrophy transformation and that any nontrivial quasigroup that is the homomorphic image of a $DC -$ quasigroup is a $DC -$ quasigroup;
3. The spectrum of finite $DC - T -$ quasigroups is characterized. It is shown that there exist $DC - T -$ quasigroups of order n, for any $n \geq 5$, $n \neq 6$;
4. Characterizations of $totCO -$ quasigroups are obtained. In particular, it is shown that the class of $totCO -$ quasigroups is closed under parastrophy transformation and homomorphic images, and necessary and sufficient conditions are given when a binary $T -$ quasigrup is a $totCO -$ quasigroup;
5. C.C. Lindner and D. Steedly [10] showed that there exist finite binary quasigroups with exactly 1, 2, 3 or 6 distinct parastrophes and completely characterized their spectrum. To solve these

problems, they used, in particular, five identities of length four, with two variables. The author of the thesis specified this result, showing that one of the five identities may be eliminated (Proposition 1.1.4);

6. Necessary and sufficient conditions are given when a ternary $T-$quasigroup has exactly k distinct parastrophes, where $k = 3, 4$ or 6. Examples are constructed and the following estimations of their spectrum are given:

   a) there exist finite ternary quasigroups, with exactly three distinct parastrophes, of any order q ≥ 3;

   b) there exist finite ternary $T-$quasigroups with exactly four distinct parastrophes, of any odd order q ≥ 3;

   c) there exist finite ternary $T-$quasigroups exactly six distinct parastrophe of any odd order $\mathrm{q}, (\mathrm{q}, 3) = 1$;

7. Necessary and sufficient conditions are given when a $4-$ary $T-$quasigrup has exactly 1, 5, 10 or 20 distinct parastrophes. Some estimations of the spectrum of such quasigroups are given;

8. It is proved that there does not exist $4-$ary $\mathrm{T}-$quasigroups with exactly 2, 6, or 15 distinct parastrophes;

9. Estimations of the spectrum of parastrophic-orthogonal and totally parastrophic-orthogonal $n-$ary quasigroups, $n = 2, 3, 4$, including of $totCO-$quasigroups are qiven:

   a) there does not exist binary $totCO-$quasigroups of order less than 7, but there exist such quasigroups of any order q, which is mutually prime with 2, 3, 5 and 7;
   b) there exist finite ternary quasigroups with exactly three distinct parastrophes, which are also orthogonal, of any odd order q ≥ 3;
   c) there exist finite ternary quasigroups of any odd order q ≥ 3 which have exactly 4 distinct parastrophes which are also orthogonal;
   d) there exist 4-ary quasigroups with exactly 5 orthogonal parastrophes of any order q ≥ 3;

10. Based on the method of constructing orthogonal systems of $n-$ary operations, given by Trevor Evans, constructions of $n-$ary parastrophic-orthogonal operations are given, in particular constructions of $k-$ary self-rthogonal operations, where k has one of the following forms: $k = n^2, 2^n, mn$, with $n, m \geq 2$. Thus it is obtained that, if on a finite set Q there exist self-orthogonal $m-$ary and, respectively $n-$ary quasigroups, where $n, m \geq 2$, then on this set there exist self-orthogonal $mn-$ary quasigroups. In particular, from this construction it

follows that there exist self-orthogonal 2^n −ary quasigroups of any order $q \neq 1, 2, 3, 6$, for every $n \geq 1$.

The results of the author T. Rotari (T. Popovich) on the topic of her doctoral thesis were published in 27 scientific papers, including 10 scientific articles and 17 abstracts of communications at scientific conferences.

Recommendations:

1. The method of characterizing $T-$quasigroups with an exact given number of distinct parastrophes can be used to obtain analogous results in the case of $4-$ary quasigroups with exactly k distinct parastrophes for $k \geq 24, k|120$;
2. Tthe spectrum of ternary quasigroups with exactly two or exactly eight distinct parastrophes requires the development of new methods for constructing such quasigroups, for example, combinatorial methods;
3. When characterizing $n-$ary totally parastrophic-orthogonal quasigroups with exactly k distinct parastrophes, where $k < n$, other definitions of orthogonality can be used;
4. Orthogonal systems of $n-$ary quasigroups, $n \geq 2$, can be used in constructing MDS-codes, in cryptography, in planning experiments, in combinatorics, etc.

BIBLIOGRAPHY

1. BELOUSOV, V. Parastrophic-orthogonal quasigroups. In: *Quasigroups and related systems,* 2005, V. 13, No. 1, pp. 25–72. ISSN 1561-2848
2. BELOUSOV, V. *Foundations of the Theory of Quasigroups and Loops*. (in Russian) Moscow: Nauka, 1967. 224 p.
3. BELYAVSKAYA, G., POPOVICH, T. On the classes of quasigroups defined by the conjugate sets. In*: Abstracts of Conference "Mathematics&Information technologies: Research and Education" (MITRE-2011),* August 22-25, 2011, Chișinău, Republic of Moldova, pp. 8-9. ISBN 978-9975-71-137-1
4. BELYAVSKAYA, G., POPOVICH, T. Totally parastrophe orthogonal quasigroups and complete graphs (in Russian). In: *Fundamentalinaya i prikladnaya mathematica*, 2010, V. 16 (8), pp. 17-26. ISSN 1560-5159
5. BELYAVSKAYA, G., POPOVICH, T. Conjugate sets of loops and quasigroups. DC-quasigroups. In: *Bul. Acad. Științe Repub. Mold. Mat.,* 2012, V. 1 (68), pp. 21-31. ISSN 1024-7696
6. BELYAVSKAYA, G., POPOVICH, T. About quasigroups with distinct conjugates. In: *Book of abstracts of the 8-th International Algebraic Conference in Ukraine Dedicated to the memory of Professor V.M. Usenko*, Lugansk, 5-12 July, 2011, p. 247.
7. BOSE, R. C., SHRIKHANDE, S. S., PARKER, E. T. Further results of the constructions of mutually orthogonal latin squares and the falsity of Euler's conjecture. In: *Canadian Journal of Mathematics*. 1960, V. 12, pp. 189-203. ISSN 0008-414X
8. DENES, J., KEEDWELL, A. D. *Latin squares and their applications*: Second edition. Elsevier, 2015. 453 p. ISBN 978-0444-63-555-6
9. FRYZ, I., SOKHATSKY, F. Construction of medial ternary self-orthogonal quasigroups. In: *Bul. Acad. Științe Repub. Mold. Mat.,* 2022, No. 3 (100), pp. 41-55. ISSN 1024-7696
10. LINDNER, C. C., STEEDLY, D. On the number of conjugates of a quasigroup. In: *Algebra Univ.,* 1975, V. 5, pp. 191–196. ISSN 0002-5240
11. McLEISH, M. On the number of conjugates of n −ary quasigroups. In*: Canadian Journal of Mathematic,* 1979, V. 31, nr. 3, pp. 637-654. ISSN 0008-414X
12. McLEISH, M. On the existence of ternary quasigroups with 2 or 8 conjugacy classes. In: *Journal of Combinatorial Theory,* Seria A, 1980, V.29, nr. 2, pp. 199-211. ISSN 0097-3165
13. MOUFANG, R. Alternativkorper und der Satz vom vollstandigen Vierseit. In: *Adhandlungen aus dem Mathetischen Seminar der Universitat Hamburg,* 1933, V. 9, pp. 207-222, ISSN

0014-4428

14. PFLUGFELDER, H. O. *Quasigroups and loops: introduction.* Berlin: Heldermann Verlag, 1990. 147 p. ISBN 978-3885-38-007-8
15. SADE, A. Quasigroupes obéissant à certaines lois. În: *Revue de la Faculté des Sciences de l'Université d'Istanbul, Série A*. 1957, V. 22, pp. 151–184. ISSN 0364-541X
16. SOKHATSKY, F., FRYZ, I. Invertibility criterion of composition of two multiary quasigroups. În: *Commentationes Mathematicae Universitatis Carolinae*. 2012, V. 53, nr. 3, pp. 429–445. ISSN 0010-2628
17. SOKHATSKY, F., PIRUS, Y. Classification of ternary quasigroups according to their parastrophic symmetry groups, I. In: *Visnik DonNU. Ser. A: Prirod. nauki*, 2018, nr. 1-2, pp. 70–81. ISSN 2522-4468
18. SOKHATSKY, F., PIRUS, Y. Classification of ternary quasigroups according to their parastrophic symmetry groups, II. In: *Visnik DonNU. Ser. A: Prirod. nauki*, 2019, nr. 1-2, pp. 101–110. ISSN 2522-4468
19. SUSCHKEWITSCH, A. K. Zur Theorie der endlichen Gruppen nichtassoziativer Systeme. In: *Mathematische Annalen*. 1928, V. 99, pp. 30–50. ISSN 0025-5831
20. SYRBU, P. On orthogonality and self-orthogonality of n-ary operations (în Rusă). In: *Mat. Issled.* Chișinău: Știința, 1987, V. 95, pp. 121–130. ISSN 0542-9994

LIST OF THE AUTHOR'S PUBLICATIONS ON THE THESIS TOPIC

a) *Articles in SCOPUS-indexed scientific journals:*

1. ROTARI, T., SYRBU, P. On 4-T-quasigroups with exactly 20 distinct parastrophes. *Bul. Acad. Ştiinţe Repub. Mold. Mat.,* 2025, No. 1 (107), pp. 107-119. ISSN 1024-7696
2. BELEAVSCAIA, G., POPOVICI, T. Near-totally conjugate orthogonal quasigroups. *Bul. Acad. Ştiinţe Repub. Mold. Mat*., 2014, nr. 3 (76), pp. 89-96. ISSN 1024-7696
3. BELYAVSKAYA, G., POPOVICH, T. Totally conjugate orthogonal quasigroups and complete graphs. *Journal of Mathematical Sciences,* 2012, vol.185, no. 2, pp. 184–191. ISSN 1072-3374
4. BELYAVSKAYA, G., POPOVICH, T. Conjugate sets of loops and quasigroups. DC-quasigroups. *Bul. Acad. Ştiinţe Repub. Mold. Mat.,* 2012, nr. 1 (68), pp. 21-31. ISSN 1024-7696
5. POPOVICH, T. On conjugate sets of quasigroups. In: *Bul. Acad. Ştiinţe Repub. Mold. Mat.*, 2011, nr. 3(67), pp. 69-76. ISSN 1024-7696
6. BELYAVSKAYA, G., POPOVICH, T. Totally parastrophe orthogonal quasigroups and complete graphs (in Russian). In: *Fundamentalinaya i prikladnaya mathematica*, 2010, V. 16 (8), pp. 17-26. ISSN 1560-5159

b) Articles in collections of articles of scientific conferences:

7. SYRBU, P., ROTARI, T. On Self-Orthogonal n-ary Quasigroups. In: *Proceedings of the Int. Conf. dedicated to the 60th anniversary of the foundation of V. Andrunachievici Institute of Mathematics and Computer Science,* October 10-13, 2024, Chişinău, Republic of Moldova, pp. 126 – 131. ISBN 978-9975-68-523-8
8. POPOVICH, T. On near-conjugate-orthogonal quasigroups. In: *Proceedings of the Third Conference of Mathematical Society of Moldova IMCS-50,* August 19-23, 2014, Chişinău, Republic of Moldova, pp. 150-153. ISBN 978-9975-68-245-9
9. BELYAVSKAYA, G., POPOVICH, T. *On graphs related to quasigroups.* Topics in Graph Theory: A tribute to A. A. and T. E. Zykovs on the occasion of A. A. Zykov's 90th birthday**.** University of Illinois at Urbana-Champaign, 2013, Illinois, SUA, pp. 187–193. Available: https://kostochk.web.illinois.edu/Zykov90-Topics_in_Graph_Theory.pdf
10. POPOVICH, T. Conjugate-orthogonal Quasigroups and Graphs. In: *International Workshop on Intelligent Information System*: *Proceeding IIS,* September 13-14, 2011, Chişinău, Republic of Moldova, pp. 256-259. ISBN 978-9975-4237-0-0

c) Abstracts of communications at international and national specialized scientific conferences:

11. ROTARI, T., SYRBU, P. On n −ary T −quasigroups with a prescribed maximum number of distinct parastrophes. In: *Abstracts of the 32nd International Conference on Applied and Industrial Mathematics (CAIM 2025),* September 18 – 21, 2025, Bucharest, Romania, p. 99. ISSN 2537-2688
12. ROTARI, T. On ternary quasigroups with exactly three distinct and orthogonal parastrophes. In: *Abstracts of the International Conference on Applied and Industrial Mathematics (CAIM 2024),* September 19-22, 2024, Oradea, Romania, p. 64. ISSN 2537-2688
13. SYRBU, P., ROTARI, T. On self-orthogonal finite n-ary quasigroups. In: *Abstracts of the International Conference on Applied and Industrial Mathematics (CAIM 2024),* September 19-22, 2024. Oradea, Romania, p. 63. ISSN 2537-2688
14. ROTARI, T. On the conjugate sets of IP-quasigroups. In: *Abstracts of the XII International Algebraic Conference in Ukraine dedicated to the 215th anniversary of V. Bunyakovsky*, July 02-06, 2019. Vinnytsia, Ukraine, pp. 94-95 Available: https://jiac.donnu.edu.ua/article/view/6993
15. POPOVICI, T., SHCHERBACOV, V. Parastrophes orthogonality of alinear quasigroups. In: *Abstracts of the XI International Algebraic Conference in Ukraine dedicated to the 75th anniversary of V. V. Kirichenko,* July 03-07, 2017, Kyiv, Ukraine, p. 104. Available: https://imath.kiev.ua/~algebra/iacu2017/abstracts_pdf
16. POPOVICH, T. On sets of conjugates of quasigroups. In: *Abstracts of the 8-th International Algebraic Conference in Ukraine Dedicated to the memory of Professor Vitaliy Mikhaylovich Usenko,* July 5-12, 2011, Lugansk, Ukraine, p. 269. Available: https://dspace.luguniv.edu.ua/xmlui/bitstream/handle/123456789/2647/_2011.pdf?sequence=1&isAllowed=y
17. BELYAVSKAYA, G., POPOVICH, T. About quasigroups with distinct conjugates. In: *Abstracts of the 8-th International Algebraic Conference in Ukraine Dedicated to the memory of Professor Vitaliy Mikhaylovich Usenko,* July 5-12, 2011, Lugansk, Ukraine, p. 247. Available: https://dspace.luguniv.edu.ua/xmlui/bitstream/handle/123456789/2647/_2011.pdf?sequence=1&isAllowed=y
18. POPOVICH, T. On parastrophe-orthogonal quasigroups and graphs (in Russian). In: *Abstracts of the 10-th International Seminar Discrete mathematics and its Aplications,* February 1-6, 2010. Moscow, Russia, pp. 258-260. Available: https://keldysh.ru/dms/10dmsem-2010.pdf
19. BELYAVSKAYA, G., POPOVICH, T. Totally conjugate-orthogonal quasigroups. In: *Abstracts of the 7-th International Algebraic Conference in Ukraine.* August 18-23, 2009. Kharkov, Ukraine, pp.

26-27. Available:
https://www.academia.edu/48370922/7th_International_Algebraic_Conference_in_Ukraine
20. ROTARI, T. Ternary quasigroups with exactly four distinct parastrophes, which are orthogonal. In: *Abstracts of the National conference with international participation: Natural Sciences in the Dialogue of Generations*, September 18-19, 2025, Chișinău, Republic of Moldova, p. 22. ISBN 978-9975-62-898-3
21. ROTARI, T., SYRBU, P. On 4-quasigroups with exactly five distinct parastrophes. In: *Abstracts of the International Conference on Quasigroups and Related Systems (ConfQRS),* July 2 – 4, 2025, Chișinău, Republic of Moldova, p. 33. ISBN 978-9975-62-880-8
22. ROTARI, T., SYRBU, P. On 4-quasigroups with exactly ten distinct parastrophes. In: *Abstracts of the International Scientific Conference "Mathematics & IT: Research and Education" (MITRE 2025),* June 26-29, 2025. Chișinău, Republic of Moldova, p. 24. ISBN 978-9975-62-879-2 (PDF)
23. ROTARI, T. Parastrophes of ternary quasigroups and their orthogonality. In: *Abstracts of the International Scientific Conference "Mathematics & IT: Research and Education" (MITRE 2023),* June 26-29, 2023. Chișinău, Republic of Moldova, p. 32. ISBN 978-9975-62-535-7
24. POPOVICH, T. Orthogonal sets of conjugates of T-quasigroups. In: *Abstracts of Conference on Applied and Industrial Mathematics Dedicated to Academician Mitrofan M. Ciobanu.* Chișinău, August 22-25, 2012. Chișinău, Republic of Moldova, pp. 187-188. ISBN 978-9975-76-090-4
25. POPOVICH, T. On the conjugate sets of quasigroups and identities. In: *Abstracts of the International Scientific Conference "Mathematics & IT: Research and Education" (MITRE 2011),* August 22-25, 2011. Chișinău, Republic of Moldova, pp. 95-96. ISBN 978-9975-144-9
26. BELYAVSKAYA, G., POPOVICH, T. On the classes of quasigroups defined by the conjugate sets. In: *Abstracts of the International Scientific Conference "Mathematics & IT: Research and Education"(MITRE 2011),* August 22-25, 2011. Chișinău, Republic of Moldova, pp. 8-9. ISBN 978-9975-144-9
27. BELYAVSKAYA, G., POPOVICH, T. On totally and near-totally conjugate-orthogonal quasigroups. In: *Abstracts of the International Scientific Conf. "Mathematics & IT: Research and Education" (MITRE 2009),* October 8-9, 2009. Chișinău, Republica of Moldova, pp. 3-4. ISBN 978-9975-70-891-3

ADNOTARE

la teza cu titlul **„Quasigrupuri cu parastrofii distincți ortogonali”,** înaintată de candidatul **Rotari Tatiana,** pentru conferirea titlului științific de doctor în științe matematice la specialitatea

111.03 – Logică matematică, algebră și teoria numerelor,

Chișinău, 2026

Structura tezei: teza este scrisă în limba română și cuprinde: introducere, patru capitole, concluzii generale și recomandări, bibliografie 162 de titluri și 5 anexe. Teza conține 112 pagini cu text de bază. Rezultatele obținute sunt publicate în 27 lucrări științifice cu volum total de circa 6,06 coli de autor.

Cuvinte-cheie: quasigrup n-ar, parastrof, quasigrup liniar, T-quasigrup, quasigrup (total) parastrofic-ortogonal, DC-quasigrup, $totCO$-quasigrup.

Scopul și obiectivele lucrării. Scopul tezei constă în obținerea unor caracterizări ale quasigrupurilor n-are ($n = 2, 3, 4$), ce posedă un număr maximal posibil de parastrofi distincți, în particular, a quasigrupurilor total parastrofic-ortogonale, precum și estimarea spectrului lor. Pentru atingerea scopului vizat sunt fixate următoarele obiective: studiul unor clase de quasigrupuri binare și n-are cu un număr dat de parastrofi distincți, inclusiv ortogonali; dezvoltarea unor metode de construcție a quasigrupurilor n-are parastrofic-ortogonale.

Noutatea și originalitatea științifică. În lucrare sunt introduse două clase noi de quasigrupuri binare: $DC-$quasigrupuri (cei șase parastrofi sunt distincți) și $totCO$-quasigrupuri (cei șase parastrofi sunt ortogonali). Problema existenței quasigrupurilor ce posedă un număr dat de parastrofi distincți și a caracterizării spectrului lor, formulată de Lindner și Steedly, este considerată pentru clasa n-quasigrupurilor liniare ($n = 2, 3, 4$), inclusiv cu sistemele maximale ortogonale de parastrofi distincți.

Problema științifică importantă soluționată constă în caracterizarea quasigrupurilor binare ce posedă 6 parastrofi distincți, respectiv 6 parastrofi ortogonali, în descrierea T-quasigrupurilor binare cu 1, 2, 3 sau 6 parastrofi distincți și a T-quasigrupurilor 4-are ce posedă numărul maximal de $1, 5, 10$ sau 20 de parastrofi distincți, inclusiv distincți și ortogonal, și estimarea spectrului lor.

Semnificația teoretică și valoarea aplicativă a lucrării. Rezultatele referitoare la T-formele n-T-quasigrupurilor cu un număr maximal dat de parastrofi distincți, inclusiv ortogonali, cât și metodele propuse de construcție a n-quasigrupurilor parastrofic-ortogonale, reprezintă contribuții la soluționarea problemelor deschise despre existența n-quasigrupurilor cu un număr dat de parastrofi distincți și spectrul n-quasigrupurilor parastrofic-ortogonale.

Implementarea rezultatelor științifice. Sistemele ortogonale de quasigrupuri $n-$are, $n \geq 2$, sunt utilizate cu succes la construirea MDS-codurilor, în criptografie, la planificarea experimentelor, în combinatorică ș.a. Rezultatele lucrării pot fi utilizate în calitate de suport pentru cursuri universitare de specialitate.

ANNOTATION

of the thesis entitled **"Quasigroups with orthogonal distinct parastrophes",** presented by the candidate **Rotari Tatiana,** for obtaining the degree of Doctor in Mathematical Sciences with specialty **111.03 Mathematical logic, algebra and number theory,**

Chisinau, 2026

Structure of the thesis: the thesis is written in Romanian and consists of an introduction, four chapters, general conclusions and recommendations, a bibliography of 162 titles and 5 appendices. The thesis contains 112 pages of basic text. The obtained results were published in 27 papers with a volume of over 6,06 sheets of author.

Keywords: n-quasigroup, parastrophe, linear quasigroup, T -quasigroup, (totally) parastrophic- orthogonal quasigroup, *DC*-quasigroup, *totCO*-quasigroup.

Research purpose and objectives: The purpose of the Thesis is to obtain characterizations of n-ary quasigroups (n=2,3,4), which possess a maximal possible number of distinct parastrophes, in particular, maximal orthogonal sets of parastrophes, as well as to estimate their spectrum. To achieve the intended goal, the following objectives are set: the study of classes of binary and n-ary quasigroups with a given number of distinct parastrophes, including orthogonal ones; the development of methods for constructing parastrophic-orthogonal n-ary quasigroups.

Scientific novelty and originality: In the present Thesis, two new classes of binary quasigroups are introduced: *DC*-quasigroups (the six parastrophes are distinct) and *totCO*-quasigroups (the six parastrophes are orthogonal). The problem of the existence of quasigroups, which possess a given number of distinct parastrophes, and of the characterization of their spectrum, formulated by Lindner and Steedly, is considered for the class of linear n-quasigroups ($n = 2, 3, 4$), including with maximal systems of orthogonal distinct parastrophes.

The result obtained: consists in characterizing binary quasigroups possessing 6 distinct parastrophes, respectively 6 orthogonal parastrophes, in the description of binary and 4-ary T-quasigroups, possessing a given maximum number 1, 2, 3 or 6, and respectively, 1, 5, 10 or 20 of distinct parastrophes, including distinct and orthogonal parastrophes, and estimating their spectrum.

The theoretical significance and applicative value: The results concerning the T-forms of n-T-quasigroups with a given maximal number of distinct parastrophes, including orthogonal ones, as well as the proposed methods for constructing parastrophic-orthogonal n-quasigroups, represent contributions to the solution of open problems about the existence of n-quasigroups with a given number of distinct parastrophes and the spectrum of parastrophic-orthogonal n-quasigroups.

Implementation of the results: Orthogonal systems of n-quasigroups, $n \geq 2$, are used in the theory of *MDS*-codes, in criptography, planning experiments, in combinatorics etc. The results may be applied as a support for teaching courses in higher education.

АННОТАЦИЯ

к диссертации **«Квазигруппы, у которых различные парастрофы ортогональны»**, представленная **Ротарь Татианой** на соискание степени доктора математических наук наук по специальности – **111.03 Математическая логика, алгебра и теория чисел,**

Кишинёв, 2026

Структура диссертации: диссертация написана на румынском языке и состоит из введения, четырех глав, общих выводов и рекомендаций, библиографии из 162 названий и 5 приложений. Диссертация содержит 112 страниц основного текста. Полученные результаты опубликованы в 27-и научных работах с общим объёмом около 6,06 авторских листов.

Ключевые слова: n-квазигруппа, парастроф, линейная квазигруппа, *T*-квазигруппа, (тотально) парастрофно-ортогональная квазигруппа, *DC*-квазигруппа, *totCO*-квазигруппа.

Цель и задачи работы: Цель диссертации состоит в описании n-квазигрупп ($n = 2, 3, 4$), обладающих заданным числом различных парастрофов, в частности, максимальным ортогональным множествам парастрофов, а также оценить их спектр. Для достижения поставленной цели определены следующие задачи: изучение классов бинарных и n-арных квазигрупп с заданным числом различных парастрофов, включая ортогональных; разработка методов построения парастрофно-ортогональных n-квазигрупп.

Научная новизна и оригинальность: В диссертации вводятся и исследуются два новых класса квазигрупп: *DC*-квазигруппы и *totCO*-квазигруппы. Проблема существования квазигрупп, обладающих заданным числом различных парастроф, и описание их спектра, сформулированная Линднером и Стедли, зассматривается для класса линейных n-квазигрупп (n=2,3,4), в том числе обладающих ортогональной системой различных парастрофов.

Решенная научная проблема: состоит в описании бинарных квазигрупп, обладающих 6 различными парастрофами, соответственно 6 ортогональными парастрофами, описании бинарных и 4-арных *T*-квазигрупп, обладающих максимальным числом из 1, 2, 3 или 6 и, соответственно, из 1, 5, 10 или 20 различных парастрофов, в том числе различных и ортогональных парастрофов, и оценки их спектра.

Теоретическое значение и прикладная ценность работы: Результаты, касающиеся T-форм n-T-квазигрупп с заданным числом различных парастрофов, включая ортогональные, а также предложенные методы построения парастрофно-ортогональных n-квазигрупп, представляют собой вклад в решение открытых проблем существования n-квазигрупп с заданным числом различных парастрофов и описания спектра парастрофно-ортогональных n-квазигрупп.

Внедрение результатов. Ортогональные системы n-квазигрупп, $n \geq 2$, успешно применяются при построении *MDS*-кодов, в криптографии, при планировании экспериментов, в комбинаторике и т.д. Результаты могут быть применены для разработки специальных курсов в системе высшего образования.

ROTARI Tatiana

QUASIGROUPS WITH ORTHOGONAL DISTINCT PARASTROPHES

111.03. Mathematical logic, algebra and number theory

Summary of the Ph.D. thesis in mathematical sciences

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